

Übungen zur Vorlesung Mathematik für Chemiker 2

Lösungen

1. (a)

$$\begin{aligned}
 \mathbb{B} &= \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, -x \leq y \leq x\} \\
 \int_B (1 + xy^2) \, dx \, dy &= \int_0^1 dx \int_{-x}^x dy (1 + xy^2) \\
 &= \int_0^1 dx \, 2\left(x + \frac{x^4}{3}\right) \\
 &= \frac{17}{15}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \mathbb{B} &= \{(x, y) \in \mathbb{R}^2 \mid -1 \leq y \leq 1, |y| \leq x \leq 1\} \\
 \int_B (1 + xy^2) \, dx \, dy &= \int_{-1}^1 dy \int_{|y|}^1 dx (1 + xy^2) \\
 &= \int_{-1}^1 dy \left(1 - |y| + \frac{y^2}{2} - \frac{y^4}{2}\right) \\
 &= 2 \int_0^1 dy \left(1 - y + \frac{y^2}{2} - \frac{y^4}{2}\right) \\
 &= \frac{17}{15}
 \end{aligned}$$

2. (a)

$$\begin{aligned}
 2x^2 - 6 = 6 - x^2 &\Leftrightarrow 3x^2 - 12 = 0 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2 \\
 &\Rightarrow S_1 = (-2, 2), \quad S_2 = (2, 2)
 \end{aligned}$$

(b) Skizze mit Parabeln und Schnittpunkt

(c)

$$\begin{aligned}\int_B 1 + \frac{2y}{x^2} dy dx &= \int_{-2}^2 \int_{2x^2-6}^{6-x^2} 1 + \frac{2y}{x^2} dx dy \\ &= \int_{-2}^2 \left[y + \frac{y^2}{x^2} \right]_{2x^2-6}^{6-x^2} dx \\ &= \int_{-2}^2 \left[6 - x^2 - 2x^2 + 6 + \frac{(6 - x^2)^2 - (2x^2 - 6)^2}{x^2} \right] dx \\ &= \int_{-2}^2 \left[12 - 3x^2 + \frac{36 - 12x^2 + x^4 - 4x^4 + 24x^2 - 36}{x^2} \right] dx \\ &= \int_{-2}^2 [12 - 3x^2 + 12 - 3x^2] dx \\ &= \int_{-2}^2 24 - 6x^2 dx \\ &= 24x - 2x^3 \Big|_{-2}^2 \\ &= 48 + 48 - 16 - 16 \\ &= 64\end{aligned}$$

3.

$$\begin{aligned}V &= \text{Volumen} \\ &= \int_B dV \\ &= \int_{z=0}^{z=1} dz \int_{y=0}^{y=1-z} dy \int_{x=0}^{x=1-y-z} dx \\ &= 3 \int_0^1 dz \int_0^{1-z} dy [x] \Big|_0^{1-z-y} \\ &= 3 \int_0^1 dz \int_0^{1-z} dy (1 - z - y) \\ &= \int_0^1 dz \left[y - zy - \frac{y^2}{2} \right]_0^{1-z}\end{aligned}$$

$$\begin{aligned} &= \int_0^1 dz \left(1 - z - z + z^2 - \frac{1}{2} + z - \frac{z^2}{2} \right) \\ &= \int_0^1 dz \left(\frac{1}{2} - z + \frac{z^2}{2} \right) \\ &= \left[\frac{z}{2} - \frac{z^2}{2} + \frac{z^3}{6} \right]_0^1 \\ &= \frac{1}{2} - \frac{1}{2} + \frac{1}{6} \\ &= \frac{1}{6} \end{aligned}$$